A.1 (Wahrheitstafeln)

a)

(i)

A	B	¬ (A ∨ B)	¬ A ∧¬ B
0	0	1	$1 \wedge 1 = 1$
0	1	0	$1 \wedge 0 = 0$
1	0	0	$0 \wedge 1 = 0$
1	1	0	$0 \wedge 0 = 0$

(ii)

A	B	¬ (A ∧ B)	¬ A ∨¬ B
0	0	1	$1 \vee 1 = 1$
0	1	1	$1 \vee 0 = 1$
1	0	1	$0 \vee 1 = 1$
1	1	0	$0 \vee 0 = 0$

(iii)

A	B	C	A ∧ (B ∨ C)	(A ∧ B) ∨ (A ∧ C)
0	0	0	0	$0 \vee 0 = 0$
0	0	1	0	$0 \vee 0 = 0$
0	1	0	0	$0 \vee 0 = 0$
0	1	1	0	$0 \vee 0 = 0$
1	0	0	0	$0 \vee 0 = 0$
1	0	1	1	$0 \vee 1 = 1$
1	1	0	1	$1 \vee 0 = 1$
1	1	1	1	$1 \vee 1 = 1$

(iv)

A	B	A ⇒ B	(¬ A ∨ B)
0	0	1	$1 \vee 0 = 1$
0	1	1	$1 \vee 1 = 1$
1	0	0	$0 \vee 0 = 1$
1	1	1	$0 \vee 1 = 1$

b)

$(A \Rightarrow B) = (\neg A \vee B) = (B \vee \neg A) = (\neg(\neg B) \vee (\neg A)) = (\neg B \Rightarrow \neg A)$

c)

$$(A \Rightarrow B) \Rightarrow C = A \Rightarrow (B \Rightarrow C) \quad (1)$$

A	B	C	(A ⇒ B) ⇒ C	A ⇒ (B ⇒ C)
0	0	0	$1 \Rightarrow 0 = 0$	$0 \Rightarrow 1 = 1$
0	0	1	$1 \Rightarrow 1 = 1$	$0 \Rightarrow 1 = 1$
0	1	0	$1 \Rightarrow 0 = 0$	$0 \Rightarrow 0 = 1$
0	1	1	$1 \Rightarrow 1 = 1$	$0 \Rightarrow 1 = 1$
1	0	0	$0 \Rightarrow 0 = 1$	$1 \Rightarrow 1 = 1$
1	0	1	$0 \Rightarrow 1 = 1$	$1 \Rightarrow 1 = 1$
1	1	0	$1 \Rightarrow 0 = 0$	$1 \Rightarrow 0 = 0$
1	1	1	$1 \Rightarrow 1 = 1$	$1 \Rightarrow 1 = 1$

(2)

Die Implikation ist nicht assoziativ.

A.2 (Mengenoperationen)

a)

$$M \cup N \qquad M \backslash N$$

(i)

$$M = \{0, 1, 2, 3\} \qquad N = \{2, 4, 6\}$$
$$M \cup N = \{0, 1, 2, 3, 4, 6\}$$
$$M \backslash N = \{0, 1, 3\}$$

(ii)

$$M = \{x \in R | 0 \leq x \leq 1\} \qquad N = \{y \in R | 1 \leq y < 2\}$$
$$M \cup N = \{z \in R | 0 \leq z < 2\}$$
$$M \backslash N = \{z \in R | 0 \leq z < 1\}$$

(iii)

$$M = \left\{x \in R | x^2 \leq 2\right\} \qquad N = \{x \in R | x < sqrt2\}$$
$$M \cup N = \left\{x \in R | x \leq \sqrt{2}\right\}$$
$$M \backslash N = \left\{x \in R | x = \sqrt{2}\right\}$$

b)

$$M \backslash N$$
$$M = \{\{1,1\},\{1,2\},\{2,1\},\{2,2\},\{3,1\},\{3,2\}\}$$
$$N = \{\{2,2\},\{2,3\},\{2,4\},\{3,2\},\{3,3\},\{3,4\},\{4,2\},\{4,3\},\{4,4\}\}$$
$$M \backslash N = \{\{1,1\},\{1,2\},\{2,1\},\{3,1\}\} = \{1,2,3\} \times \{1\}$$

c)

$$\text{Seien: A = 1, B = 3, C = 2 \& D = 3}$$
$$\{\{1\} \times \{3\}\} \setminus \{\{1\} \times \{3\}\} = \{\{1\} \setminus \{2\}\} \times \{\{3\} \setminus \{3\}\}$$
$$\{1,3\} \setminus \{1,3\} = \{1\} \setminus \{\}$$
$$\emptyset = \{1\}$$

Die allgemeine Gleichheit der Terme ist nicht gegeben.

d)

(i)

$$M = \{a; b; c\}$$
$$P(M) = \{\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\},\emptyset\}$$

(ii)

$$M = \emptyset$$
$$P(M) = \emptyset$$

(iii)

$$P(M) = \{\{1\},\{\{2\}\},\{1,\{2\}\},\emptyset\}$$

A.3 (Zurückführung von Regeln der Mengenoperationen auf Regeln der Aussagenlogik)

a)

$$A \cap B = \underbrace{\{a | a \in A \wedge a \in B\}}_{Kommutativgesetz}$$

b)

$$(A \cap B) \cup C =$$
$$= \{a|\,(a \in A \wedge a \in B) \vee a \in C\} = \underbrace{\{a|\,(a \in A \vee a \in C) \wedge (a \in B \vee a \in C)\}}_{Distributivgesetz} =$$
$$= (A \cup C) \cap (B \cup C)$$

A.4 (Quantoren)

1. Wahr
2. Falsch
3. Falsch
4. Falsch
5. Wahr
6. Wahr
7. Falsch